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MR1163728 (93e:32048) 32S45 (14B05 32S25)

de Jong, Theo (D-KSRL); **van Straten, Duco** (D-KSRL)

A construction of Q-Gorenstein smoothings of index two.

Internat. J. Math. **3** (1992), *no. 3,* 341–347.

A conjecture of 1144527J. Kollar [in Surveys in differential geometry (Cambridge, MA, 1990), 113–199, Lehigh Univ., Bethlehem, PA, 1991; MR1144527 (93b:14059)] explains smoothing components of rational surface singularities by deformations of partial resolutions, on which only singularities with qG-smoothings occur; a one-parameter smoothing $X_T \to T$ of a surface singularity, Gorenstein in codimension 1, is qG if some multiple of the canonical class of X_T is Cartier, and the smallest value is the index. This motivates the problem of which singularities have qG-smoothings.

The authors prove that a smoothing is qG if and only if it is $\omega^{[1-r]}$-constant (r is the index), meaning that the restriction map $\omega_{X_T}^{[1-r]} \otimes \mathcal{O}_X \to \omega_X^{[1-r]}$ is surjective. Here $\omega_X^{[k]} = (\omega_X^{\otimes k})^{**}$ for $k > 0$ and $\omega_X^{[k]} = \mathrm{Hom}_X(\omega_X^{[-k]}, \mathcal{O}_X)$ if $k < 0$. In the case $r = 2$ this condition is related to the number of triple points in a generic projection [MR 92d:32050T. de Jong and D. van Straten, Math. Ann. 288 (1990), no. 3, 527–547; MR1079877 (92d:32050)]. As a corollary of the theory of their earlier paper [op. cit.], the authors obtain that a rational quadruple point of index 2 has a qG-smoothing, a result found by the reviewer, using the classification of quadruple points [MR 91m:14006Internat. J. Math. 2 (1991), no. 2, 205–221; MR1094706 (91m:14006)].

The construction of qG-smoothings is based on the following setup: let X_1 and X_2 be two surfaces with projections $Y_i = \{f_i = 0\}$ with the same double curve $\Sigma = V(I)$. Suppose that $\int I/I^2$ is a cyclic $\mathcal{O}$-module. The authors prove that there is a one-to-one correspondence between $\omega^{[-1]}$-constant smoothing components of X_1 and X_2. If (f_1, f_2) defines a multiplicity four structure on Σ, these components are qG. By taking f_1 as a product of linear forms the authors find a series of examples of rational surface singularities with one qG-component.

Reviewed by *Jan Stevens*

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